

Curriculum vitae et studiorum

Bruno Franchi

Name: Franchi, Bruno.

Born: Reggio Emilia, April 4th, 1950.

Home address: via Filippo Ferrari 3, 42121 Reggio Emilia, Italy.

Citizenship: Italian.

Personal data: married with Spallanzani, Mariafranca, born in Scandiano (Reggio Emilia, Italy), April 12th, 1950, Italian citizen, professor at the University of Bologna. One daughter: Franchi, Caterina, born in Reggio Emilia, August 5th, 1985, Italian citizen, student.

Academic degrees: Laurea in Mathematics (University of Bologna, Italy - November 21st, 1973).

Current position:

- ◆ From May 2022 professor emeritus of the Alma Mater University of Bologna;
- ◆ Since July 2018 corresponding member of the Academy of Sciences of the Istituto di Bologna.

Previous positions in Italy:

- ◆ from November 2020 until May 2022 professor of the Alma Mater University of Bologna;
- ◆ from 1990 until 2020 full professor of Mathematical Analysis at the Department of Mathematics of the University of Bologna (SSD MAT/05 - Analisi Matematica, Settore Concorsuale 01/A3 - Analisi Matematica, Probabilità e Statistica Matematica);
- ◆ From 1987 until 1990 full professor of Mathematical Analysis at the University of Torino, Italy;
- ◆ From 1980 until 1987 associate professor of Mathematical Analysis at the University of Bologna, Italy;
- ◆ From 1974 until 1980 researcher and then teaching assistant at the University of Bologna;
- ◆ In 1974-75 military service.

Foreign positions:

- ◆ In 1985-86 for a semester associate teacher l'Université de Paris Sud (Orsay);
- ◆ In 1994-95 member of the Institute of Advanced Study of Princeton (NJ - USA).

Main research topics:

- i) Degenerate elliptic and parabolic equations.
- ii) Homogenization.
- iii) Geometric measure theory in non-Euclidean structures.
- iv) Methods of real analysis for integral inequalities in metric spaces.
- v) Differential geometry in nilpotent groups.
- vi) Mathematical models in medicine (role of proteins in Alzheimer's disease).

Further international experiences:

- ◆ In 1997 visiting professor at Rutgers University, NJ - USA (one month).

- ◆ In 2000 visiting professor at the Université de Cergy Pontoise, France (one month).
- ◆ In 2001 visiting professor at Université de Reims, France (one month).
- ◆ In 2005 member of the MSRI of Berkeley (6 weeks).
- ◆ In 2006 visiting professor at Université de Paris VII-Jussieu (one month).
- ◆ In 2010 visiting professor at Pittsburgh University (2 weeks).
- ◆ In 2016 visiting professor at the Université de Paris-Sud (one month).
- ◆ In 2017 visiting professor at the Universitat Politècnica de Catalunya (Barcelona) (2 weeks).
- ◆ Visiting professor for shorter periods at: Ecole Polytechnique, Université de Paris-Sud, Leiden University, Institut Fourier (Grenoble), Aalto University (Helsinki), Rutgers University, Université de Paris VII-Jussieu, Univ. Autonoma of Madrid, University of Sevilla, Bern University, Universitat Politècnica de Catalunya, Bergen University, Princeton University, Polish Academy of Sciences, Academy of Sciences of the Czech Republic, Wayne State University, Beijing Normal University, Wuhan University, Trondheim University (Norway), EPF de Lausanne, Université de Nice (France)
- ◆ In 1983 visiting fellow at the Ecole Polytechnique (Palaiseau - France) (3 months).
- ◆ In 1988, 1989, 1990, 2016 visiting fellow at the Université de Paris Sud (Orsay) (3 weeks every year).
- ◆ In 1991 visiting fellow at Princeton University (one month).
- ◆ In 2016 visiting fellow at Rutgers University, NJ - USA (2 weeks).

Scientific organization of schools, meetings and seminars:

- ◆ Scientific co-organizer of 11 *Schools on Analysis and Geometry in Metric Spaces* (currently held in Levico Terme).
- ◆ Scientific co-organizer of the *Conference in Harmonic Analysis and applications in honor of Dick Wheeden* (Sevilla 2010).
- ◆ Scientific co-organizer of the workshop *Geometric methods in pde's* (Bologna 2008).
- ◆ Scientific co-organizer of the school *Four mini courses on fine properties of solutions of Partial Differential Equations* (Pisa - Centro De Giorgi 2006).
- ◆ Scientific co-organizer of the workshop *Heat Kernels, Stochastic Processes and Functional Inequalities* (Oberwolfach 2005).
- ◆ Scientific co-organizer of the meeting *Integral inequalities and applications* (Cortona 1999).
- ◆ I gave many talks within international meetings and at mathematical institutions in Italy and abroad.

Among the most recent talks:

- (1) *Poincaré and Sobolev inequalities for differential forms in Euclidean spaces and Heisenberg groups* (Imperial College in London 2020)

- (2) *Poincaré and Sobolev inequalities for differential forms in Heisenberg groups* (Centro De Giorgi - SNS Pisa 2020)
- (3) *Poincaré and Sobolev inequalities for differential forms in Heisenberg groups (in collaboration with A. Baldi and P. Pansu)* (Jyväskylä 2019)
- (4) *Poincaré and Sobolev inequalities for differential forms in Heisenberg groups (in collaboration with A. Baldi and P. Pansu)* (Baltimore 2019)
- (5) *Poincaré and Sobolev inequalities for differential forms in Euclidean spaces and Heisenberg groups* (Parix XI - Orsay 2019)
- (6) *Poincaré and Sobolev inequalities for differential forms in Euclidean spaces and Heisenberg group* (IHP Paris 2019)
- (7) *Rectifiability, regular manifolds and differential forms in Carnot groups* (Univ. Autonoma di Madrid November 2017)
- (8) *Sobolev-Poincaré inequalities for differential forms in Heisenberg groups* Univ. Helsinki (May 2017)
- (9) *A mathematical model for onset and progression of Alzheimer disease* (In memory of J. Serrin, Perugia January 2017)
- (10) *Poincaré inequalities for differential forms on Heisenberg groups* (EPF Lausanne, 2018)
- (11) *Alzheimer's disease: a mathematical model for onset and progression* (Paris, LJLL 2018)
- (12) *Gagliardo-Nirenberg inequalities for differential forms in Heisenberg groups* (Barcelona 2015)
- (13) *Gagliardo-Nirenberg inequalities for differential forms in Heisenberg groups* (Padova 2016)
- (14) *Gagliardo-Nirenberg inequalities for differential forms in Heisenberg groups* (Bologna, Accademia delle Scienze 2016)
- (15) *Smoluchowski equation with variable coefficients in perforated domains: homogenization and applications to mathematical models in medicine* (Catania 2016)
- (16) *Differential forms in Heisenberg groups and div-curl systems* (Princeton University 2014)
- (17) *Differential forms in Heisenberg groups and div-curl systems* (FJIM Bilbao, 2014)
- (18) *Lectures on differential forms in Carnot groups* (Bergen 2014)
- (19) *The role of beta amyloid in Alzheimer disease: a mathematical model* (Mittag-Leffler Institute 2013)
- (20) *Sharp a priori estimates for div-curl systems in Heisenberg groups* (IHP- Paris 2013)
- (21) *A diffusion-coagulation equation for the concentration of beta-amyloid in Alzheimer's disease* (Parma 2012)
- (22) *Maxwell's equations in Carnot groups* (Beijing Normal University, Beijing 2012)
- (23) *Maxwell's equations in Carnot groups* (Genova 2012)
- (24) *Maxwell's equations in Carnot groups* (Cortona 2012)
- (25) *Diffusione e agglomerazione della beta-amiloide* (Torino 2012)
- (26) *Maxwell's equations in Carnot groups* (Univ. Autonoma Madrid 2011)
- (27) *Maxwell's equations in Carnot groups* (Aalto University, Helsinki 2010)
- (28) *Maxwell's equations in Carnot groups* (Pittsburgh, 2010)
- (29) *Maxwell's equations in Carnot groups* (Pisa, 2010)
- (30) *Differential forms in Carnot groups: a variational approach* (Sevilla 2010)
- (31) *Compensated compactness in Carnot groups* (Bologna 2008)
- (32) *Lipschitz graphs within Heisenberg groups* (Roma 2008)
- (33) *Compattezza per compensazioni in gruppi di Carnot* (Milano 2008)
- (34) *Compensated compactness, div-curl theorem and H -convergence in general Heisenberg groups* (Cortona 2006)

Research evaluation:

- ◆ Member of the Editorial Board of the journal *Analysis and Geometry in Metric Spaces*, de Gruyter publisher.
- ◆ In 2010 member of the Panel for Research Proposals of the Research Council for Natural Sciences and Engineering of Academy of Finland.
- ◆ In 2000 member of the panel for the evaluation ex-post of national research projects.

- ◆ Referee for projects of the National Science Foundation (USA).
- ◆ Referee for projects of the SNS di Pisa.
- ◆ Referee for projects of the Università Statale of Milano.
- ◆ Referee for projects of the University of Padova.
- ◆ Co-advisor of a PhD student at the Università Statale of Milano.
- ◆ Member of panels or referee for PhD theses at the Universities of Trento, Padova, Parix XI - Orsay, and SNS of Pisa.
- ◆ Member of panels for the Habilitation à diriger des Recherches at the Université de Cergy-Pontoise and Université de Rennes.
- ◆ Referee for two habilitations at the University of Bern.
- ◆ Referee for tenures at several US Universities. .
- ◆ Member of the panel for a position of associate professor at the University of Bergen.
- ◆ Member of the panels for several positions of full professor, associate professor and researcher in Italian Universities and for tenures.
- ◆ Referee for several international mathematical journals.

Teaching activity:

Since 1/1/78 I taught on a regular basis courses of Mathematical Analysis for students of Mathematics, Physics, Astronomy, Engineering and Computer Sciences, as well as advanced courses for student of Mathematics (Calculus of Variations, Function Theory and Elements of Higher Analysis). In 2012-13 I taught a course for the PhD program in Mathematics of the University of Bologna on *Differential forms in Carnot groups*

Institutional activities at the University of Bologna:

- ◆ Member of the *Osservatorio della Ricerca dell'Università di Bologna* for the evaluation of the research (2008-2013).
- ◆ From 2016 until 2020 president of the Scientific Commette of the Library “Biblioteca Interdipartimentale di Matematica, Fisica, Astronomia e Informatica”.
- ◆ Coordinator of the PhD program of the University of Bologna from 2002 until 2006.
- ◆ President or member of the panel of the Department of Mathematics for the evaluation of the research of the University of Bologna and for the distribution of local research funds (RFO).
- ◆ Advisor of 9 PhD theses at the University of Bologna (5 in co-advisorship with European partners)¹
- ◆ Member of the Giunta of the Department of Mathematics.
- ◆ Coordinator of a project EC-People (with U. di Berna, Siviglia, Arkansas, Temple (Philadelphia), Pittsburgh, Illinois).

¹Annalisa Baldi (2000), Francescopaolo Montefalcone (2004), Davide Barbieri (2008) (co-advisorship (cotutela) with T. Coulhon of the Université de Cergy-Pontoise), Cristina Imperato (2008), Eleonora Cinti (2010) (co-advisorship (cotutela) with X. Cabré of the Politecnico of Catalunya), Valentina Penso (2017) (co-advisorship (cotutela) with Z. Balogh of the University of Bern), Veronica Tora (2019) (co-advisorship (cotutela) with M. Herrero of the Universidad Complutense of Madrid), Marta Marulli (2019) (co-advisorship (cotutela) with N. Vauchelet of the Université de Paris 13) Francesca Corni (the thesis will be defended in May 2021).

- ◆ Coordinator of a project Azioni Integrate Italy-Spain and of projects Galileo of the Université Italo-Française.
- ◆ Coordinator of projects for local research funds (RFO) and of a strategic project of the University of Bologna..
- ◆ Member of panels or referee for PhD theses at the University of Bologna.
- ◆ Advisor of several undergraduate theses at the University of Bologna.

Bibliometric data:

- ◆ According to MathSciNet of the American Mathematical Society: 122 publications and 2159 quotations.
- ◆ According to Google Scholar: more than 4000 citations and h-index 29.
- ◆ According to Scopus: h-index 19

Bibliometric data:

- ◆ According to the MathSciNet dell' American Mathematical Society 128 publications e 2696 quotations (August 2023).
- ◆ According to Google Scholar 5030 quotations and h-index 30 (May 2023).

LIST OF PUBLICATIONS

- [1] B. Franchi, *Spazi di Banach e forme quadratiche quasi non singolari*, Boll. Un. Mat Ital. **10** (1974), 440–450.
- [2] B. Franchi, *Su un teorema di R.H. Martin jr.*, Rend. Sem. Mat. Univ. Padova **55** (1976), 275–288.
- [3] A. Bove, B. Franchi & E. Obrecht, *An Initial-Boundary Value Problem with Mixed Lateral Conditions for Heat Equation*, Ann. Mat. Pura Appl. (4) **121**, 277–307.
- [4] A. Bove, B. Franchi & E. Obrecht, *Elliptic Equations with Polynomially Growing Coefficients in a Half Space*, Boll. Un. Mat. Ital. **17-B** (1980), 823–834.
- [5] A. Bove, B. Franchi & E. Obrecht, *A Boundary Value Problem for Elliptic Equations with Polynomial Coefficients in a Half Space I: Pseudodifferential Operators and Function Spaces*, Boll. Un. Mat. Ital. **18-B** (1980), 25–45.
- [6] A. Bove, B. Franchi & E. Obrecht, *A Boundary Value Problem for Elliptic Equations with Polynomial Coefficients in a Half Space II: The Boundary Value Problem*, Boll. Un. Mat. Ital. **18-B** (1980), 355–380.
- [6a] A. Bove, B. Franchi & E. Obrecht, *An Elliptic Boundary Value Problem with Unbounded Coefficients in a Half Space*, Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. (8) **65** (1978), 265–268.
- [7] A. Bove, B. Franchi & E. Obrecht, *Boundary Value Problems for Operators like $\Delta + x \cdot \nabla$* , Rend. Mat. (7) **1** (1981), 95–120.
- [7a] A. Bove, B. Franchi & E. Obrecht, *Elliptic Equations Containing the Term $r\partial r$ in a Half Space*, Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. (8) **67** (1979), 223–226.
- [8] A. Bove, B. Franchi & E. Obrecht, *Parabolic Problems with Mixed Time Dependent Lateral Conditions*, Comm. Partial Differential Equations **7** (1982), 1253–1288.
- [9] A. Bove, B. Franchi & E. Obrecht, *Parabolic Problems in Weighted Sobolev Spaces*, Ricerche Mat. **31** (1982), 45–68.
- [10] A. Bove, B. Franchi & E. Obrecht, *Boundary Value Problems with Mixed Lateral Conditions for Parabolic Operators*, Ann. Mat. Pura Appl. (4) **131** (1982), 375–413.
- [10a] A. Bove, B. Franchi & E. Obrecht, *Problèmes aux limites avec des conditions laterales de type mêlé pour des opérateurs paraboliques*, Seminaire d'Analyse, Université de Nantes (1980/81).
- [11] A. Bove, B. Franchi & E. Obrecht, *Straightening of a Noncylindrical Region and Evolution Equations*, Rend. Sem. Mat. Univ. Padova **71** (1984), 207–216.
- [12] B. Franchi, E. Lanconelli, *Une métrique associée à une classe d'opérateurs elliptiques dégénérés*, Rend. Sem. Mat. Univ. e Politec. Torino, Proceedings of the meeting "Linear Partial and Pseudo Differential Operators", Fascicolo Speciale (1982).
- [13] B. Franchi, E. Lanconelli, *An Embedding Theorem for Sobolev Spaces Related to Non-Smooth Vector Fields and Harnack Inequality*, Comm. Partial Differential Equations **9** (1984), 1237–1264.
- [14] B. Franchi, E. Lanconelli, *Hölder regularity Theorem for a Class of Linear Non Uniformly Elliptic Operators with Measurable Coefficients*, Ann. Scuola Norm. Sup. Pisa (4) **10** (1983), 523–541.
- [14a] B. Franchi, E. Lanconelli, *De Giorgi's Theorem for a Class of Strongly Degenerate Elliptic Equations*, Atti Accad. Naz. Lincei, Cl. Sci. Fis. Mat. Natur. (8) **72** (1982), 273–277.
- [15] B. Franchi, *Trace Theorems for Anisotropic Weighted Sobolev Spaces in a Corner*, Math. Nachr. **127** (1986), 25–50.
- [16] B. Franchi, *Propriétés des courbes intégrales de champs de vecteurs et estimations ponctuelles d'équations elliptiques dégénérées*, séminaire Goulaouic-Meyer-Schwartz 1983-1984, Exposé n. 3.
- [17] B. Franchi, E. Lanconelli, *Une condition geometrique pour l'inegalité de Harnack*, J. Math. Pures Appl. **64** (1985), 237–256.
- [18] B. Franchi, E. Lanconelli & J. Serrin, *Existence and Uniqueness of Ground States Solutions of Quasilinear Elliptic Equations*, Nonlinear Diffusion Equations and their Equilibrium States, Springer, 1988.
- [18a] B. Franchi, E. Lanconelli & J. Serrin, *Esistenza e unicità degli stati fondamentali per equazioni ellittiche quasilineari*, Atti Accad. Naz. Lincei, Cl. Sci. Fis. Mat. Natur. (8) **79** (1985), 212–216.

- [19] B. Franchi, E. Lanconelli, *Radial Symmetry of the Ground States for a Class of Quasilinear Elliptic Equations*, Nonlinear Diffusion Equations and their Equilibrium States – Berkeley 1986, Springer, 1988.
- [20] B. Franchi, R. Serapioni, *Pointwise Estimates for a Class of Strongly Degenerate Elliptic Operators: a Geometrical Approach*, Ann. Scuola Norm. Sup. Pisa (4) **14** (1987), 527–568.
- [21] B. Franchi, *Weighted Sobolev-Poincaré Inequalities and Pointwise Estimates for a Class of Degenerate Elliptic Equations*, Trans. Amer. Math. Soc. **327** (1991), 125–158.
- [22] B. Franchi, *Global Solutions for a Class of Monge-Ampère Equations*, Nonlinear Diffusion Equations and their Equilibrium States II – Gregynog 1989, Birkhäuser, 1992.
- [23] M. Bertsch, R. Dal Passo & B. Franchi, *A degenerate Parabolic Equation in Noncylindrical Domains*, Math. Ann. **294** (1992), 551–587.
- [24] B. Franchi, *Inégalités de Sobolev pour des champs de vecteurs lipschitziens*, C. R. Acad. Sci. Paris – Ser. A **311** (1990), 329–332.
- [25] B. Franchi, N. Kutev & S. Polidoro, *Nontrivial Solutions for Monge-Ampère Type Operators in Convex Domains*, Manuscripta Math. **79** (1993), 13–26.
- [26] B. Franchi, F. Serra Cassano, *Régularité partielle pour une classe de systèmes elliptiques dégénérés*, C. R. Acad. Sci. Paris, Ser. A **316** (1993), 37–40.
- [27] B. Franchi, C. Gutierrez & R.L. Wheeden, *Weighted Sobolev-Poincaré inequalities for Grushin type operators*, Comm. Partial Differential Equations **19** (1994), 523–604.
- [28] B. Franchi, S. Gallot & R.L. Wheeden, *Sobolev and isoperimetric inequalities for degenerate metrics*, Math. Ann. **300** (1994), 557–571.
- [28a] B. Franchi, S. Gallot & R.L. Wheeden, *Inégalités isopérimétriques pour des métriques dégénérées*, C. R. Acad. Sci. Paris, Ser. A **317** (1993).
- [29] B. Franchi, E. Lanconelli & J. Serrin, *Existence and uniqueness of Nonnegative Solutions of Quasilinear Elliptic Equations in \mathbf{R}^n* , Advances in Math. **118** (1996), 177–243.
- [30] B. Franchi, F. Serra Cassano, *Gehring’s lemma for metrics and higher integrability of the gradient for minimizers of noncoercive variational functionals*, Studia Math. **120** (1996), 1–22.
- [31] B. Franchi, C. Gutierrez & R.L. Wheeden, *Two-weight Sobolev-Poincaré inequalities and Harnack inequality for a class of degenerate elliptic operators*, Atti Accad. Naz. Lincei, Cl. Sci. Fis. Mat. Natur. (9), **5** (1994), 167–175.
- [32] J.C Fernanes, B. Franchi, *Existence and properties of the Green function for a class of degenerate parabolic equations*, Rev. Mat. Iberoamericana **12** (1996), 491–525.
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- [34] B. Franchi, G. Lu & R.L. Wheeden, *Representation formulas and weighted Poincaré inequalities for Hörmander vector fields*, Ann. Inst. Fourier **45** (1995), 577–604.
- [35] B. Franchi, G. Lu & R.L. Wheeden, *Weighted Poincaré inequalities for Hörmander vector fields and local regularity for a class of degenerate elliptic equations*, Potential Analysis **4** (1995), 361–375.
- [36] B. Franchi, G. Lu & R.L. Wheeden, *A relationship between Poincaré type inequalities and representation formulas in metric spaces*, Int. Math. Res. Not. (1996), 1–14.
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- [39] B. Franchi, R.L. Wheeden, *Compensation couples and isoperimetric estimates for vector fields*, Colloquium Math. **74** (1997), 9–27.
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- [46] B. Franchi, C. Pérez & R.L. Wheeden, *Sharp geometric Poincaré inequalities for vector fields and non-doubling measures*, Proc. Lond. Math. Soc. (3) **80** (2000), 665-689.
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- [52] F. Ferrari, B. Franchi, *A local doubling formula for the harmonic measure associated with subelliptic operators and applications*, Comm. Partial Differential Equations **28** (2003), 1?-60.
- [53] F. Ferrari, B. Franchi, *Geometry of the boundary and doubling property of the harmonic measure for Grushin type operators*, Proceedings of the meeting "Partial Differential Operators" (Torino, 2000), Rend. Sem. Mat. Univ. Politec. Torino **58** (2002), 281?-299.
- [54] A. Baldi, B. Franchi & G. Lu, *An existence result for degenerate elliptic pde's*, Ricerche Mat., special issue in honor of E. De Giorgi **49**, **suppl.** (2000), 177-193..
- [55] B. Franchi, M.C. Tesi, *Two-scale homogenization in the Heisenberg group*, J. Math. Pures Appl. (9) **81** (2002), 495?-532.
- [56] A. Baldi, B. Franchi, *A Γ -convergence result for doubling metric measures and associated perimeters*, Calc. Var. Partial Differential Equations, **16** (2003), 283?-298.
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- [62] A. Baldi, B. Franchi, *Mumford-Shah type functionals associated with doubling metric measures*, Proc. Roy. Soc. Edinburgh Sec A **135** (2005), 1-23.
- [63] F. Ferrari, B. Franchi & H. Pajot, *The geometric traveling salesman problem in the Heisenberg group*, Rev. Mat. Iberoamericana **23** (2007), 437-480.
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