Alejandro Vargas

alejandro@vargas.page
website: https://vargas.page/
Dipartamento di Matematica, Alma Mater Studiorum – Università di Bologna, Italy

Research interests

Main: tropical geometry; combinatorial structures in algebra and geometry, e.g. graphs, matroids, posets, Bruhat-Tits buildings; programming and computational problems.
Secondary: polytopes, extremal combinatorics, number theory.

Academic experience

2023 – 2024 **Postdoc**, *Alma Mater Studiorum – Università di Bologna, Italy*
Mentor: Luca Moci
Topics: combinatorial and topological properties of hyperplane arrangements, matroids, polimatroids, combinatorial Hodge theory, log-concavity, Kazhdan-Lusztig polynomials, and symmetric functions.

2022 – 2023 **Postdoc**, *Goethe-Universität Frankfurt in Germany, and Université de Nantes in France*
SNF project 200142.
Mentors Martin Ulirsch and Erwan Brugallé
Topics: indexed branched covers of posets and polyhedral spaces, real tropical geometry, ribbon graphs, enriched counts, valuated matroids, Bruhat–Tits buildings

2016 – 2022 **PhD, Mathematics**, *University of Bern*, Switzerland
Advisor: Jan Draisma
Thesis title: Gonality of metric graphs and Catalan-many tropical morphisms to trees
Topics: chip-firing, gonality of metric graphs, counting tropical morphisms, tropical geometry
Mention: insigni cum laude

2014–2016 **Master of Science, Mathematics**, *University of Groningen*, The Netherlands
Topics: algebra, number theory, algebraic geometry and combinatorics
Mention: cum laude

2010–2013 **Licenciatura en Matemática**, *Universidad de San Carlos*, Guatemala
Topics: Pure and applied mathematics
Mention: cum laude

Articles

in preparation
• *Polyhedral geometry of tropical solution sets associated to ODE's*
with Stefano Mereta
• *A real gonality of metric graphs*
with Erwan Brugallé
• *Catalan-many tropical morphisms to trees; Part II: A space and a count*

preprints
• 7. *Combinatorics of higher dimensional tropical covers*
arXiv: 2305.03220
• 6. *One-quasihomomorphisms from the integers into symmetric matrices*
arXiv: 2302.01611
with Tim Seynnaeve and Nafie Tairi

published
• 5. *Buildings, valuated matroids, and tropical linear spaces*
Journal of the London Mathematical Society 109, no. 1 (2024): e12850.
arXiv: 2304.09146
with Luca Battistella, Kevin Kühn, Arne Kuhrs, and Martin Ulirsch.

• 4. *On the gonality of metric graphs*
Notices of the AMS, May 2021, Pages 687 - 695
with Jan Draisma.
• 3. *Catalan-many tropical morphisms to trees; Part I: Constructions*
Journal of Symbolic Computation, Volume 104, May–June 2021, Pages 580-629
arXiv: 1909.12924
with Jan Draisma
• 2. *A short note on Cayley-Salmon equations*
Le Matematiche, Vol 75 No 2 (2020), Pages 559-574
arXiv: 1912.01464
with Marvin Hahn and Sara Lamboglia.
• 1. *Group structures on families of subsets of a group*
Pi Mu Epsilon Journal, Vol. 14, No. 5 (Fall 2016), pp. 323-326
arXiv: 1604.01119
with Mario Gómez, Sergio López-Permouth, Fernando Mazariegos and Rigoberto Zelada)

Talks

Valuated Matroids, Tropicalized Linear Spaces and the Affine Building of $PGL_{r+1}(K)$, *research talk*
2023, April: Seminar on Nonlinear Algebra of Max Planck Institute for Mathematics, Leipzig.
2023, May: Latin American Geometria Algebraica Real y TrOpical Seminar, Online.

Catalan-many tropical morphisms, *research talk*
2017, August: Participant talk at the Stockholm Master Class in Mathematics in Tropical Geometry.
2017, December: Bern-Fribourg-Neuchâtel intercity seminar, Neuchâtel.
2018, February: Mittag-Leffler institute, semester on tropical geometry, Stockholm.
2019, July: MEGA (Effective Methods in Algebraic Geometry), Madrid.
2020, April: TGiF (Tropical Geometry in Frankfurt), Online.
2022, May: University of Bern, doctoral defense
2022, August: Young Reseachers' Conference on Non-Archimedean and Tropical Geometry.
2023, April: Geometry and topology colloquium at the University of Nantes.

3 proofs that every degree-3 smooth surface in $\mathbb{P}^3_{\mathbb{C}}$ **has 27 lines**, *introductory talk*
2022, April: Bern-Fribourg graduate seminar

Counting with tropical geometry, *introductory talk*
2019, November: 20th Annual Swiss Graduate Colloquium, Genève.
2019, November: ETH Geometry graduate colloquium, Zurich.

Chip-firing and a Riemann-Roch Formula for Graphs, *introductory talk*
2016, June: Master thesis defense, University of Groningen.
2016, September: Colloquium of the department of mathematics, Universidad de San Carlos, Guatemala.
2017, January: Swiss-french workshop on algebraic geometry, Charmey.

Non-periodic Tessellations, *introductory talk*
2015, March: Seminar of master students of mathematics, University of Groningen.
2016, April: Virtual event of Guatemalan scientists.

La labor de un matemático, *general audience talk*
2018, January: Fondo de cultura Económica, Guatemala.

Selected research visits and schools

Full list at `https://vargas.page/pastconferences/`

April, 2023 **4th Graduate Student Meeting on Applied Algebra and Combinatorics**, *KTH Stockholm*, Presented a poster on 1-quasihomomorphisms

April, 2023 **Research visit**, *MPI:MiS Leipzig*, Working with Stefano Mereta on the topic of matroids in tropical differential equations. Also gave a talk.

August, 2022 **Young Reseachers' Conference on Non-Archimedean and Tropical Geometry**, *University of Regensburg*, Gave a talk about the construction of the moduli space of tropical morphisms

July, 2019 **SIAM AG 19**, *University of Bern*, co-organized a session on chipfiring and presented a poster on Catalan-many tropical morphisms to trees

July, 2018 **A Tropical Panorama**, *MPI for Mathematics in the life sciences Leipzig*, received travel support

June, 2018 **GAeL: Géométrie Algébrique en Liberté**, *University of Strassbourg*, received travel support

April, 2018 **Tropical Geometry meets Representation Theory**, *University of Cologne*, presented a poster on Catalan-many tropical morphisms to trees

Jan-Feb, 2018 **Tropical geometry, amoebas, and polytopes**, *Mittag-Leffler Institute*, invited for a research stay and to give a talk

August, 2017 **Stockholm Master Class in Tropical Geometry**, *Department of Mathematics Stockholm University*, received travel support

Teaching Experience

2023 **Organizer of reading group on tropical linear series**, *Online*
Online reading group for recent developments on tropical linear series, with collaborators from Frankfurt university and Kaiserslautern university.

01.11.2016–31.12.2020 **Teaching Assistant**, *University of Bern*, Switzerland
Graded homework and exams, held tutorials and exercise hours, for courses in: applied mathematics, algebra, algebraic number theory, algorithms in algebra, analysis, mathematical topics for biology, mathematics of data science, linear algebra, and representation theory.

01.04.2012 – 31.05.2012 **Substitute teacher**, *Colegio Capouilliez*, Guatemala
Mathematics substitute teacher for students in last year of high school. Substituted for one quarter.

10.01.2010–31.07.2014 **Math Olympiad Trainer**, *Guatemalan Mathematical Association*, Guatemala
Organize and teach training for high school students to participate in International Math Olympiads. Prepare exams to select the teams. Travel as leader of the delegation to help prepare and grade the competition exams.

20.01.2010–31.07.2014 **Teacher at a math circle**, *Colegio Capouilliez*, Guatemala
Organize activities and problem-solving sessions for several groups for children ages 12 to 17. Participation in the program was elective. Students with a perceived affinity for mathematics were strongly encouraged to participate. Some students would go on to participate in math olympiads and obtain good results.

Awards and Recognitions

Jun 2022 **Swiss national science foundation grant number 200142**, For financing 18 months postdoc

Nov 2019 **Birkhäuser prize for best talk**, 20th Annual Swiss Graduate Colloquium

Aug 2014 **Erasmus Mundus, Eurica Scholarship**, For master studies in the university of Groningen

Jun 2013 **Recognition by the Ministry of Education of Guatemala**, For preparing the students that participated during 2013 in international math olympiads

Jul 2012 **XIX International Math Competition for University Students**, Second Prize

Jul 2009 **International Math Olympiad**, Honorific Mention

Other skills

Refereeing: Experience refereeing for Selecta Mathematica.
Programming: Perl, Python, Sage, C, Javascript, Mathematica, PHP.

Languages

Spanish	Native	
English	Fluent	
Italian	B1 level	*Common European Framework of Reference for Languages*
French	B1 level	*Common European Framework of Reference for Languages*
German	B1 level	*Common European Framework of Reference for Languages*

Topics of research

Below I describe the research projects I have undertaken, with possible future outlooks.
1. **Topic:** Buildings, valuated matroids, and tropicalizations
Description: A building $\mathcal{B}$ is a cell complex with a lot of symmetry constructed in such a way that a chosen group G acts on it with a very transitive action [2]. The geometric feature of transitivity translates to being able to study the algebraic structure of G via the geometric properties of the building. These objects are particularly rich for reductive algebraic groups over non-archimedean fields K. e.g. the special linear group $SL_r(K)$, the projective linear group $PGL_r(K)$, the orthogonal linear group $O_r(K)$ and the symplectic group $Sp_r(K)$. Here the spaces are known as Affine buildings, or Bruhat-Tits buildings, because they arise from gluing infinitely many copies of a certain tiling of Euclidean space. They were constructed in the work of Bruhat and Tits [1, 3], who moreover prove them to be contractible and akin to metric spaces of non-positive curvature. By the work of [12, 16, 20], compactifications of the affine buildings associated with the groups mentioned above can be found inside the Berkovich analytification of certain varieties.
On the other hand, it is also known that Berkovich analytification [14] may be regarded as the limit of tropicalizations [9, 17]. A tropicalization is a procedure that takes an algebraic variety defined over a non-archimedean field K, and associates to it a polyhedral complex. This process is non-canonical, thus the limit taken above amounts to gluing the tropicalizations over all possible choices. Since both processes we have described so far bear a strong resemblance, a natural question is the following:
Question: does a restriction of the choices of what tropicalizations are taken enable to recover the affine buildings?
Own research: In joint work with Battistella, Kuehn, Kuhrs and Ulirsch [31], we show the question has an affirmative answer for the case of the projective linear group $PGL_r(K)$. The key insight comes from work by [4], where under the rather strong assumption that K is discretely valued (this, in particular, implies that K is not algebraically closed), they recover the building associated with $PGL_r(K)$ by gluing certain polyhedral spaces that come from so-called tight-spans of valuated matroids. These tight spans were introduced in the 90's; in the context of tropical geometry were developed into *tropical linear spaces* [10]. This suggests restricting to tropicalizations of linear spaces that are linearly embedded in projective space, which gives valuated matroids. And indeed, an intuitive description of our results is that valuated matroids approximate the desired building, we describe how each matroid sits in the building, and that the limit of all of them recovers the whole space. Moreover, our assumptions on K are mild, thus vastly generalizing previous results.
Future directions: The work in [31] initiates a program that can be continued for the other above-mentioned groups. This is an ambitious undertaking, because the projective linear group is associated with valuated matroids, and other groups give rise to other Coxeter matroids. Thus a theory of valuated Coxeter matroids calls to be developed.

2. **Topic:** Tropical differential equations
Description: In [21] Grigoriev applied tropical methods to investigate the supports of formal power series that solve a given system of ordinary differential equations. This is extended by Garay and collaborators to systems of partial differential equations in [26]. Both results work under the assumption

that K is trivially valued. In [32], Mereta extends to non-trivial valuation, and applies the framework to give results on the radius of convergence of p-adic differential equations. The main combinatorial challenge in this area is the passage to infinite combinatorial structures, namely matroids supported over an infinite ground set. Several aspects of the theory of tropical varieties are beginning to be worked out in this infinite setting, e.g. [28], but a lot of work remains to be done.
Future directions In a recently initiated collaboration with Garay and Mereta, we set to expand the combinatorial theory of T-ODE's and T-PDE's. Our first goal is to define polyhedral spaces associated with a system, i.e. the Bergman fan of a possibly infinite matroid. Preliminary investigations suggest that some equations display finite behaviour, while others display infinite behaviour. Longer term, we wish to develop algorithms for several problems of interest, e.g. a vertex finding algorithm that is relevant in the T-PDE setting, and to apply these results to specific equations of interest.

3. **Topic:** Tropical curves, tropical morphisms, and higher-dimensional tropical covers
Description: Quoting [5], "It is well-known that a finite graph can be viewed, in many respects, as a discrete analogue of a Riemann surface". The metric graphs studied by tropical geometry enrich this analogy with an additional metric structure [18, Sections 1.3, 1.7]. The tropical analogue of a holomorphic map of compact Riemann surfaces is called a *tropical morphism* $\Phi : \Gamma \to \Delta$. This is a continuous map that is piecewise linear of integral slopes, such that using the slope as a multiplicity gives that the count of points in a fibre is locally constant. Moreover, a Riemann-Hurwitz condition is satisfied, namely that the associated ramification divisor $K_\Gamma - \Phi^* K_\Delta$ is effective.
Note that the classical behaviour of holomorphic maps has to be imposed by definition on our combinatorial maps. Thus, a lot of the research in this area centres around which combinatorial axioms, so-called *realizability conditions*, have to be imposed so the combinatorial picture faithfully reflects the classical geometry. Once this goal is achieved, correspondence theorems can be proven. As an example, we mention the correspondence of classical Hurwitz numbers and tropical Hurwitz numbers [13, 11].
Own research: My research in this area started with a combinatorial study of the gonality of metric graphs. This notion is an analogue of the following classical question: given a compact Riemann surface X, what is the minimum number of poles that a non-constant meromorphic function $f : X \to \mathbb{C} \cup \{\infty\}$. This number is called the *gonality* of X. It is known, by Brill-Noether theory, that this number is $\lceil g/2 \rceil + 1$. In the even-case $g = 2g'$ there are Catalan-many $\frac{1}{g'+1}\binom{2g'}{g'}$ functions with the minimum number of poles, up to isomorphism. The tropical analogue of such functions are maps from metric graphs to trees, and the count of poles is equal to the degree of such maps, namely the count with multiplicity of points in a fibre. It was proven in [6], using heavy machinery of algebraic geometry, that the same gonality upper bound $\lceil g/2 \rceil + 1$ holds tropically. From the combinatorial point of view, such proof is dissatisfying, since the statement is purely combinatorial. In joint work with Jan Draisma [25], we give a combinatorial proof of the gonality bound. In the second part of my PhD thesis [30], an analogous result to the Catalan count is proven, which tropically was not previously known. The proof required to develop a combinatorial framework for the study of indexed branched covers [34], which is of independent interest to apply in other enumerative problems.
Future directions: In joint work with Erwan Brugallé, we are exploring how to give the tropical morphisms a real structure, i.e. a ribbon structure on top of the metric graphs plus some compatibility conditions, and perform similar counts. The main obstacle is the definition of the multiplicity of a real tropical morphism, which has been an open problem in the literature for the last few years [19].

4. **Topic:** Tropical linear series
Description: Given a smooth algebraic curve X over $\mathbb{C}$, there is a correspondence between rank-r line bundles and maps $X \to \mathbb{P}^r$. This correspondence breaks spectacularly in the tropical world, and it has been the work of many researchers to understand this failure and to find combinatorial conditions to remedy it. The failure boils down to two critical points:
• In the tropical divisor theory, as in [8] and [7], there are way more divisors than the ones coming from tropical morphisms. The super-abundance of divisors is so big that the discrepancies happen at a dimension level, see [22, 24].
• It is not clear what a tropical linear series is, namely a space of tropical rational functions since the

naive guess has really bad behaviour, including the fact that as a polyhedral complex, it is not pure dimensional [15].
On the latter point, recent work by Jensen and Payne [29], and independent work by Amini and Gierczak [27], have taken the first steps towards resolving the second difficulty, by introducing new realizability conditions that can potentially resolve the problems associated to the tropical divisor theory.
Future directions: Together with several collaborators we are working on questions of finite generation of the proposed tropical linear series, and maps to tropical linear spaces. For the latter, it is needed to investigate the matroidal properties of tropical modules.

5. Topic: Miscellanea
Description: I have also looked at questions related to tropical convexity, quasi-homomorphisms [33] and classical constructions of algebraic geometry [23].

Bibliography

[1] François Bruhat and Jacques Tits. "Groupes Réductifs Sur Un Corps Local: I. Données Radicielles Valuées". In: *Publications Mathématiques de l'IHÉS* 41 (1972), pp. 5–251.

[2] Jacques Tits and Ralph Duncan James. "On Buildings and Their Applications". In: Proceedings of the International Congress of Mathematicians. Canadian Mathematical Congress, 1975, pp. 209–220. ISBN: 0-919558-04-6.

[3] François Bruhat and Jacques Tits. "Groupes Réductifs Sur Un Corps Local: II. Schémas En Groupes. Existence d'une Donnée Radicielle Valuée". In: *Publications Mathématiques de l'IHÉS* 60 (1984), pp. 5–184.

[4] Andreas Dress and Werner Terhalle. "The Tree of Life and Other Affine Buildings". In: Proceedings of the International Congress of Mathematicians. Vol. 3. 1998, pp. 565–574.

[5] Matthew Baker and Serguei Norine. "Riemann–Roch and Abel–Jacobi Theory on a Finite Graph". In: *Advances in Mathematics* 215.2 (2007), pp. 766–788. ISSN: 0001-8708. arXiv: 0608360v3.

[6] Matthew Baker. "Specialization of Linear Systems from Curves to Graphs". In: *Algebra & Number Theory* 2.6 (2008), pp. 613–653. ISSN: 1944-7833. arXiv: 0701075v4.

[7] Andreas Gathmann and Michael Kerber. "A Riemann–Roch Theorem in Tropical Geometry". In: *Mathematische Zeitschrift* 259.1 (2008), pp. 217–230. ISSN: 0025-5874.

[8] Grigory Mikhalkin and Ilia Zharkov. "Tropical Curves, Their Jacobians and Theta Functions". In: *Curves and abelian varieties* 465 (2008), pp. 203–230.

[9] Sam Payne. "Analytification Is the Limit of All Tropicalizations". In: *arXiv preprint arXiv:0805.1916* (2008). arXiv: 0805.1916.

[10] David E Speyer. "Tropical Linear Spaces". In: *SIAM Journal on Discrete Mathematics* 22.4 (2008), pp. 1527–1558. ISSN: 0895-4801.

[11] Renzo Cavalieri, Paul Johnson, and Hannah Markwig. "Tropical Hurwitz Numbers". In: *Journal of Algebraic Combinatorics* 32.2 (2010), pp. 241–265. ISSN: 0925-9899.

[12] Bertrand Rémy, Amaury Thuillier, and Annette Werner. "Bruhat-Tits Theory from Berkovich's Point of View. I. Realizations and Compactifications of Buildings". In: Annales Scientifiques de l'Ecole Normale Supérieure. Vol. 43. 3. 2010, pp. 461–554.

[13] Benoît Bertrand, Erwan Brugallé, and Grigory Mikhalkin. "Tropical Open Hurwitz Numbers". In: *Rendiconti del Seminario Matematico della Università di Padova* 125 (2011), pp. 157–171. ISSN: 0041-8994. arXiv: 1005.4628v3.

[14] Vladimir G Berkovich. *Spectral Theory and Analytic Geometry over Non-Archimedean Fields*. 33. American Mathematical Soc., 2012. ISBN: 0-8218-9020-4.

[15] Christian Haase, Gregg Musiker, and Josephine Yu. "Linear Systems on Tropical Curves". In: *Mathematische Zeitschrift* 270.3-4 (2012), pp. 1111–1140. ISSN: 0025-5874.

[16] Bertrand Remy, Amaury Thuillier, and Annette Werner. "Bruhat–Tits Theory from Berkovich's Point of View. II Satake Compactifications of Buildings". In: *Journal of the Institute of Mathematics of Jussieu* 11.2 (2012), pp. 421–465. ISSN: 1475-3030.

[17] Jeffrey Giansiracusa and Noah Giansiracusa. "The Universal Tropicalization and the Berkovich Analytification". In: (Oct. 16, 2014). URL: https://arxiv.org/abs/1410.4348v2 (visited on 01/30/2019).

[18] Diane Maclagan and Bernd Sturmfels. *Introduction to Tropical Geometry*. Vol. 161. American Mathematical Soc., 2015. ISBN: 0-8218-5198-5.

[19] Hannah Markwig and Johannes Rau. "Tropical Real Hurwitz Numbers". In: *Mathematische Zeitschrift* 281.1-2 (2015), pp. 501–522. ISSN: 0025-5874.

[20] Bertrand Rémy, Amaury Thuillier, and Annette Werner. "Bruhat-Tits Buildings and Analytic Geometry". In: *Berkovich spaces and applications* (2015), pp. 141–202. ISSN: 3319110284.

[21] Dima Grigoriev. "Tropical Differential Equations". In: *Advances in Applied Mathematics* 82 (2017), pp. 120–128. ISSN: 0196-8858.

[22] Martin Moeller, Martin Ulirsch, and Annette Werner. "Realizability of Tropical Canonical Divisors". In: *arXiv preprint arXiv:1710.06401* (2017). arXiv: 1710.06401.

[23] Marvin Anas Hahn, Sara Lamboglia, and Alejandro Vargas. "A Short Note on Cayley-Salmon Equations". In: *arXiv preprint arXiv:1912.01464* (2019). arXiv: 1912.01464.

[24] Martin Ulirsch and Dmitry Zakharov. "Tropical Double Ramification Loci". In: *arXiv preprint arXiv:1910.01499* (2019). arXiv: 1910.01499.

[25] Jan Draisma and Alejandro Vargas. "Catalan-Many Tropical Morphisms to Trees; Part I: Constructions". In: *Journal of Symbolic Computation* (2020). ISSN: 0747-7171.

[26] Sebastian Falkensteiner, Cristhian Garay-López, Mercedes Haiech, Marc Paul Noordman, Zeinab Toghani, and François Boulier. "The Fundamental Theorem of Tropical Partial Differential Algebraic Geometry". In: Proceedings of the 45th International Symposium on Symbolic and Algebraic Computation. 2020, pp. 178–185.

[27] Omid Amini and Lucas Gierczak. "Limit Linear Series: Combinatorial Theory". In: *arXiv preprint arXiv:2209.15613* (2022). arXiv: 2209.15613.

[28] Alex Fink and Zeinab Toghani. "Initial Forms and a Notion of Basis for Tropical Differential Equations". In: *Pacific Journal of Mathematics* 318.2 (2022), pp. 453–468. ISSN: 1945-5844.

[29] David Jensen and Sam Payne. "Tropical Linear Series and Tropical Independence". In: *arXiv preprint arXiv:2209.15478* (2022). arXiv: 2209.15478.

[30] Alejandro Vargas. "Gonality of Metric Graphs and Catalan-many Tropical Morphisms to Trees". Thesis. Universität Bern, 2022. DOI: 10.48549/3855. URL: https://boristheses.unibe.ch/3855/.

[31] Luca Battistella, Kevin Kuehn, Arne Kuhrs, Martin Ulirsch, and Alejandro Vargas. "Buildings, Valuated Matroids, and Tropical Linear Spaces". In: *arXiv preprint arXiv:2304.09146* (2023). arXiv: 2304.09146.

[32] Stefano Mereta. "The Fundamental Theorem of Tropical Differential Algebra over Nontrivially Valued Fields and the Radius of Convergence of Nonarchimedean Differential Equations". In: *arXiv preprint arXiv:2303.12124* (2023). arXiv: 2303.12124.

[33] Tim Seynnaeve, Nafie Tairi, and Alejandro Vargas. "One-Quasihomomorphisms from the Integers into Symmetric Matrices". In: *arXiv preprint arXiv:2302.01611* (2023). arXiv: 2302.01611.

[34] Alejandro Vargas. "Combinatorics of Higher-Dimensional Tropical Covers". In: *arXiv preprint arXiv:2305.03220* (2023). arXiv: 2305.03220.